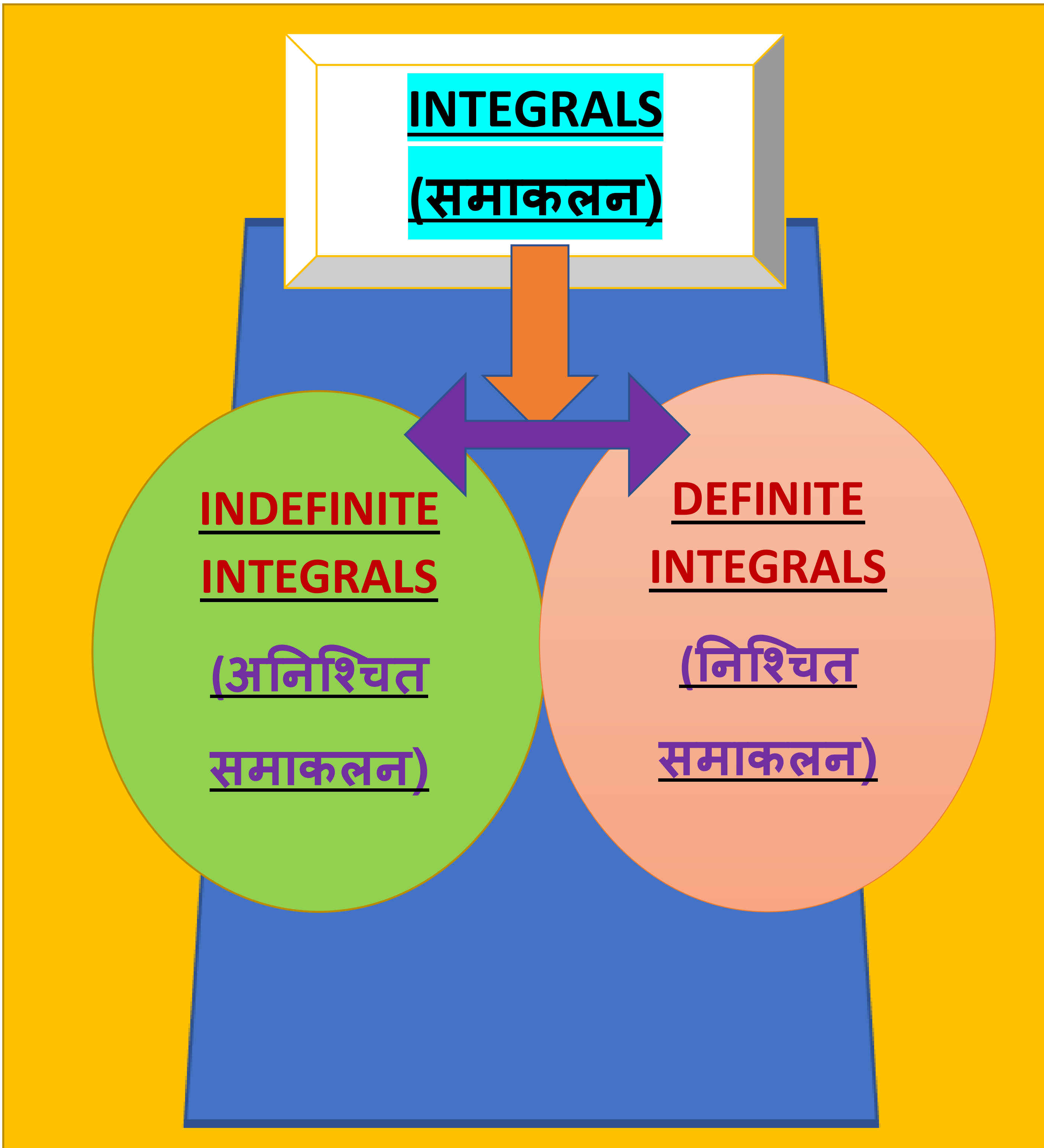
CLASS-12



INTEGRALS
(समाकलन)
INDEFINITE INTEGRALS
(अनिश्चित समाकलन)
DEFINITE INTEGRALS
(निश्चित समाकलन)

INTRODUCTION:

The development of integral calculus arises out of the efforts of solving the problems of the following types :

(a)The problem of finding a function whenever its derivative is given.

यदि एक फलन का अवकलज ज्ञात हो तो उस फलन को ज्ञात करने की समस्या।

(b) The problem of finding the area bounded by the graph of a function under certain condition.

निश्चित प्रतिबंधों के अंतर्गत फलन के आलेख से घिरे क्षेत्र का क्षेत्रफल ज्ञात करने की समस्या ।

The definite integral is used to solve many interesting problems from various disciplines like economics, finance, probability and is a practical tool for science and engineering.

Integration as the inverse process of differentiation(समाकलन को अवकलन के व्युत्क्रम प्रक्रम के रूप में): Integration is the inverse process of differentiation instead of differentiating a function, we are given the derivative of a function and asked to find its primitive (पूर्वग), that is, the original function (वास्तविक फलन) such a process is called integration (समाकलन) or anti differentiation (प्रति अवकलन).

- d/dx (sinx)= cosx
- d/dx (x^3/3)= 1/3 d/dx (x^3)=1/3 × 3x^2 = x^2
- d/dx (e^x)= e^x

- d/dx (sin x + C)= d/dx (sin x) + d/dx ©
  = cos x+0=cos x
- d/dx (x^3/3 +C)= d/dx (x^3/3) + d/dx (C)= x^2
- d/dx (e^x+ C) = e^x , where C is any real number treated as constant function.
- Here cos x is derivative of function sin x in other words sin x is an antiderivative or an integral of cos x .Similarly x^3/3 is an integral of x^2 and e^x is an integral of e^x . Again, integral of cos x is (sin x + C) that indicates in integrals are not unique.

$\int \cos x \, dx = \sin x$

- $d/dx (\sin x + C) = \cos x$,

where C=constant of Integration.

$\int \cos x \, dx = \sin x + C$

- $d/dx (\sin x + 3) = \cos x$

$\int \cos x \, dx = \sin x + 3$

- $d/dx (\sin x + \sqrt{7}) = \cos x$

$\int \cos x \, dx = \sin x + \sqrt{7}$

- Note: $d/dx [F(x) + C] = f(x)$

$\int f(x) \, dx = F(x) + C$

- $dy/dx = f(x)$

$y = \int f(x) \, dx$

Symbols/ Terms/Phrases	Meaning
$\int f(x)\,dx$	Indefinite integral of f w.r.t. x
f(x) in $\int f(x)\,dx$	Integrand (समाकल्य)
x in $\int f(x)\,dx$	Variable of integration
Constant of Integration	Any real number C, considered as constant function

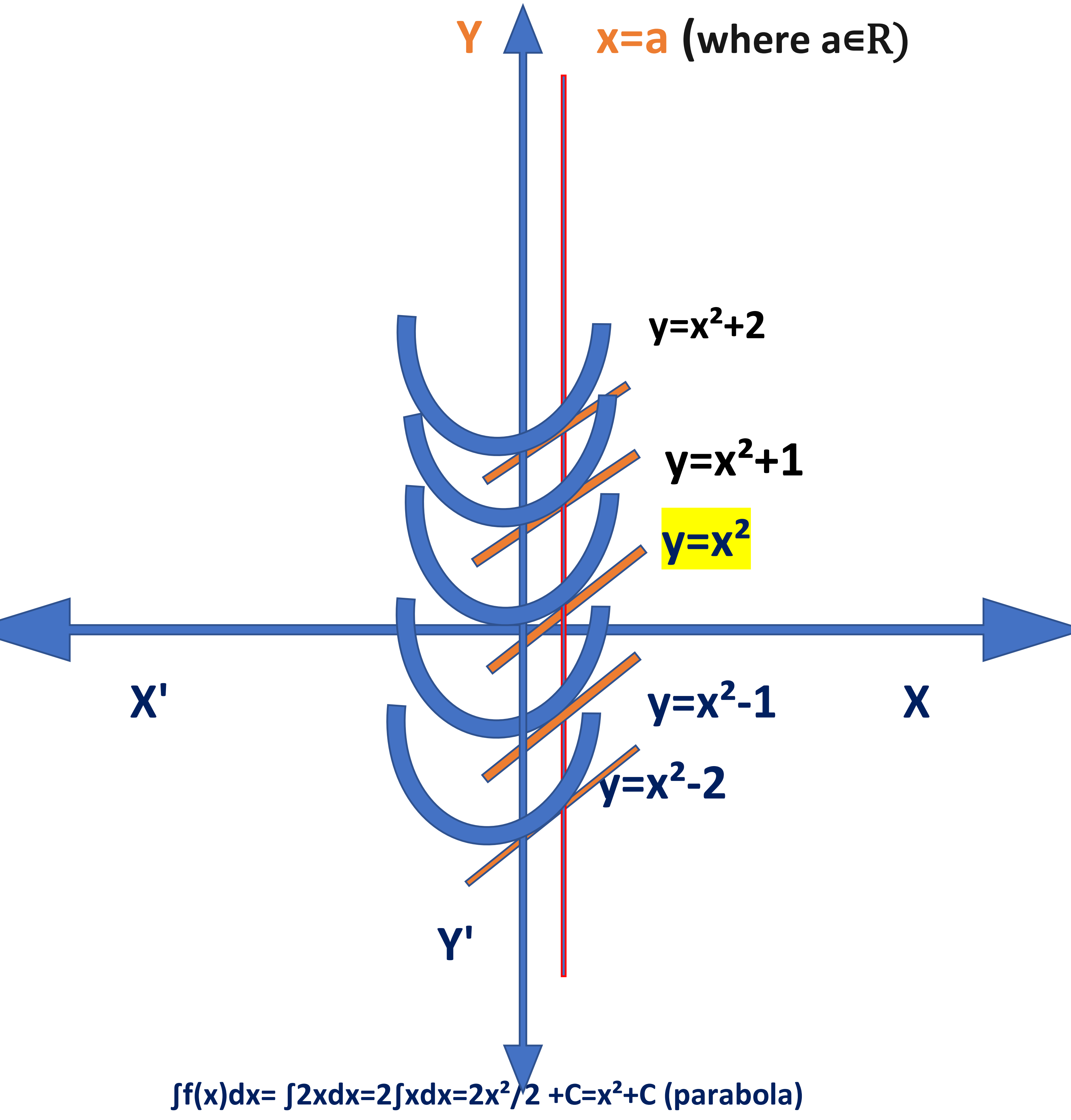
Note : d/dx ∫ f(x)= f(x)

∫ f' (x) dx= f (x) + C, where C is any arbitarary constant .(स्वेच्छ अचर)

Use	For finding	When we know that
Derivative	Velocity	Distance transversed at any time t
Integral	Distance	Velocity at time t

- **Geometrical interpretation of indefinite integral :**

A collection of of family of curves,each of which is obtained by translating one of the curves parallel to itself upwards or downwards along the y- axis.



$\int f(x)dx = \int 2xdx = 2\int xdx = 2x^2/2 + C = x^2 + C$ (parabola)

*STANDARD FORMULAE

DERIVATIVES	INTEGRALS (ANTI DERIVATIVES)
$d/dx\ (x^{n+1}/n+1)=x^n$, $n \neq -1$	$\int x^n dx = x^{n+1}/n+1\ + C$, $n \neq -1$
$d/dx\ (x)=1$	$\int dx = x+C$ & $\int af(x)dx = a\int f(x)dx$
$d/dx\ (\sin x)=\cos x$	$\int \cos x\ dx=\sin x+C$
$d/dx\ (\cos x)= -\sin x$	$\int \sin x\ dx= -\cos x+C$
$d/dx\ (\tan x)= \sec^2 x$	$\int \sec^2 x\ dx=\tan x+C$
$d/dx\ (-\cot x) = \mathrm{cosec}^2\ x$	$\int \mathrm{cosec}^2\ x\ dx=-\cot x+C$
$d/dx\ (\sec x) = \sec x.\tan x$	$\int \sec x.\tan x\ dx=\sec x+C$
$d/dx\ (-\mathrm{cosec}\ x) =\mathrm{cosec}\ x.\mathrm{sec}x$	$\int \mathrm{cosec}\ x\ .\cot x\ dx=-\mathrm{cosec}\ x+C$
$d/dx\ (\sin^{-1} x)=1/\sqrt{1-x^2}$	$\int dx/\sqrt{1-x^2}= \sin^{-1}\ x+C$
$d/dx\ (\cos^{-1} x)=-1/\sqrt{1-x^2}$	$\int -1/\sqrt{1-x^2}\ dx= \cos^{-1} x+C$
$d/dx\ (\tan^{-1} x)=1/1+x^2$	$\int \cos x\ dx=\sin x+C$
$d/dx\ (\cot^{-1} x)=-1/1+x^2$	$\int -1/1+x^2\ dx= \cot^{-1} x+C$
$d/dx\ (\log\lvert x\rvert)=1/x$, $x \neq 0$	$\int 1/x\ dx=\log\ \lvert x\rvert+C$, $x \neq 0$

$\frac{d}{dx}(a^x/\log a) = a^x$	$\int a^x\,dx = a^x/\log a + C$
	$\int \tan x\,dx = \log\lvert\sec x\rvert + C$ $= -\log\lvert\cos x\rvert + C$
	$\int \cot x\,dx = \log\lvert\sin x\rvert + C$
	$\int \sec x\,dx$ $= \log\lvert\sec x + \tan x\rvert + C$
	$\int \text{cosec}\,x\,dx$ $= \log\lvert\text{cosec}\,x - \cot x\rvert$ $= -\log\lvert\text{cosec}\,x + \sec x\rvert + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}(\sec^{-1} x)$ $= 1/x\sqrt{(x^2-1)}$	$\int dx/x\sqrt{(x^2-1)}$ $= \sec^{-1} x + C$
$\frac{d}{dx}(\text{cosec}^{-1} x)$ $= -1/x\sqrt{x^2-1}$	$\int -dx/x\sqrt{(x^2-1)}$ $= \text{cosec}^{-1} x + C$

$\int dx/(x^2-a^2)=1/2a \quad \log|x-a/x+a|+C$

$\int dx/(a^2-x^2)=1/2a \quad \log|a+x/a-x|+C$

$\int dx/(x^2+a^2)=1/a \quad \tan^{-1} x/a \ +C$

$\int dx/ \sqrt{(x^2-a^2)}= \ \log\ |x+\sqrt{x^2-a^2}|+C$

$\int dx/\sqrt{(a^2-x^2)}= \sin^{-1}\ x/a + C$

$\int dx/\sqrt{(x^2+a^2)}= \log\ |x+\sqrt{x^2+a^2}|+C$

$\int \sqrt{(x^2-a^2)}\ dx$

$=x/2\ \sqrt{(x^2-a^2)} - a^2/2 \log\ |x+\sqrt{x^2-a^2}|+C$

$\int \sqrt{(a^2-x^2)}\ dx$

$=x/2\ \sqrt{(a^2-x^2)} + a^2/2 \sin^{-1} x/a + C$

$\int \sqrt{(x^2+a^2)}\ dx$

$=x/2\ \sqrt{(x^2+a^2)}+ a^2/2 \log\ |x+\sqrt{(x^2+a^2)}|+ C$