DIRECTION-COSINES OF A LINE

- **CLASS-**XII
- **SUBJECT-**MATHEMATICS
- **CHAPTER-**3-D GEOMETRY
- **TOPIC-**DIRECTION-COSINES
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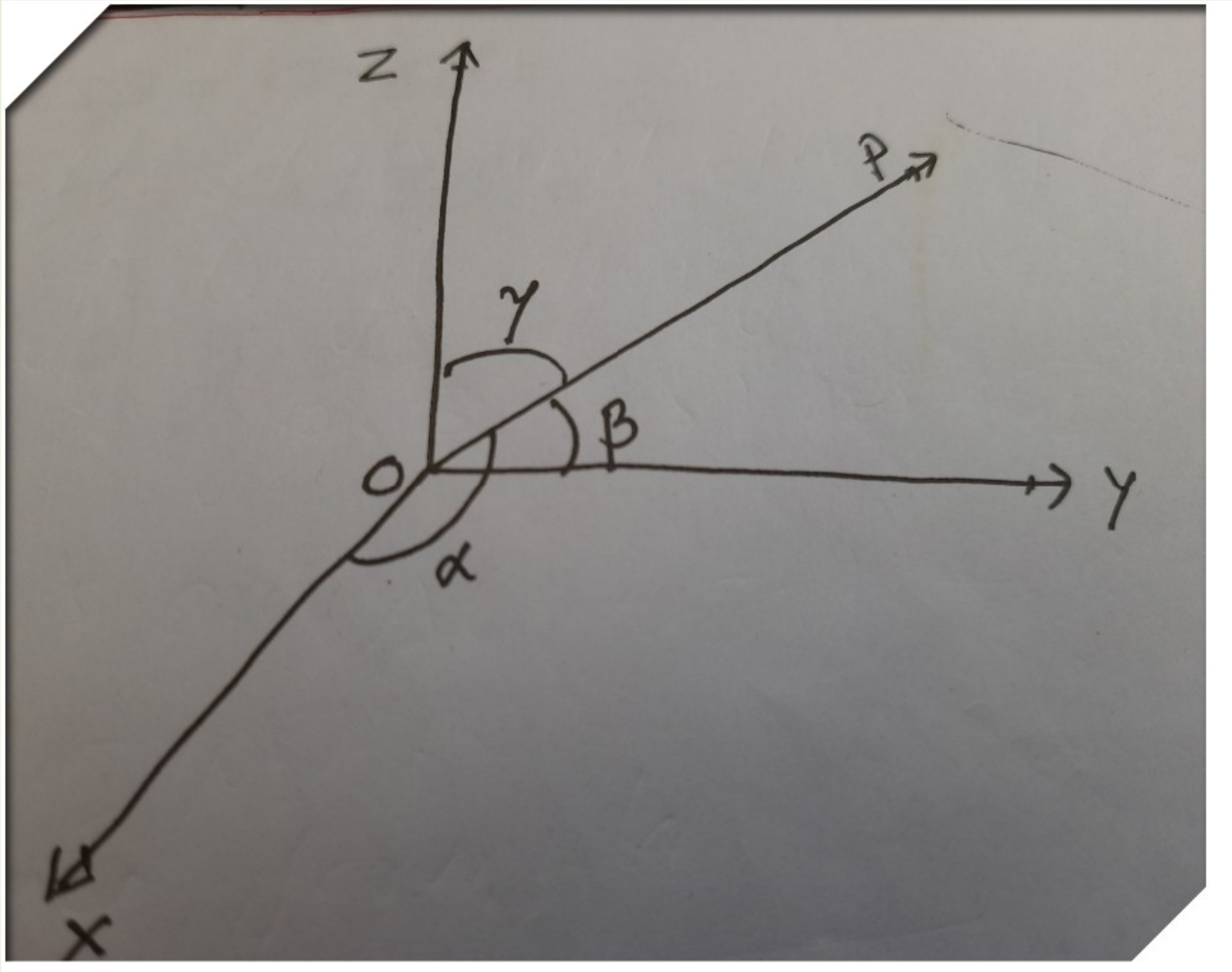


Let OP= l be a line.

Let l be a line which makes angles $\propto, \beta$ and γ $with$ the $positive$ x, y and z − axis respectively. Then $\cos\alpha, \cos\beta, \cos\gamma$ are $called$ $direction - cosines$ of a $line$ l. $Generally$, If is $denoted$ by l, m, n.

Here, l= $\cos\alpha$, m=cosβ,n=cosγ.

Question-If a line makes angles of 90°,60° and30° with the positive x, Y and z-axis respectively. Find its Direction – Cosines.

Solution- Given,

a=90°, $\beta = 60°, \gamma = 30°$

Now,

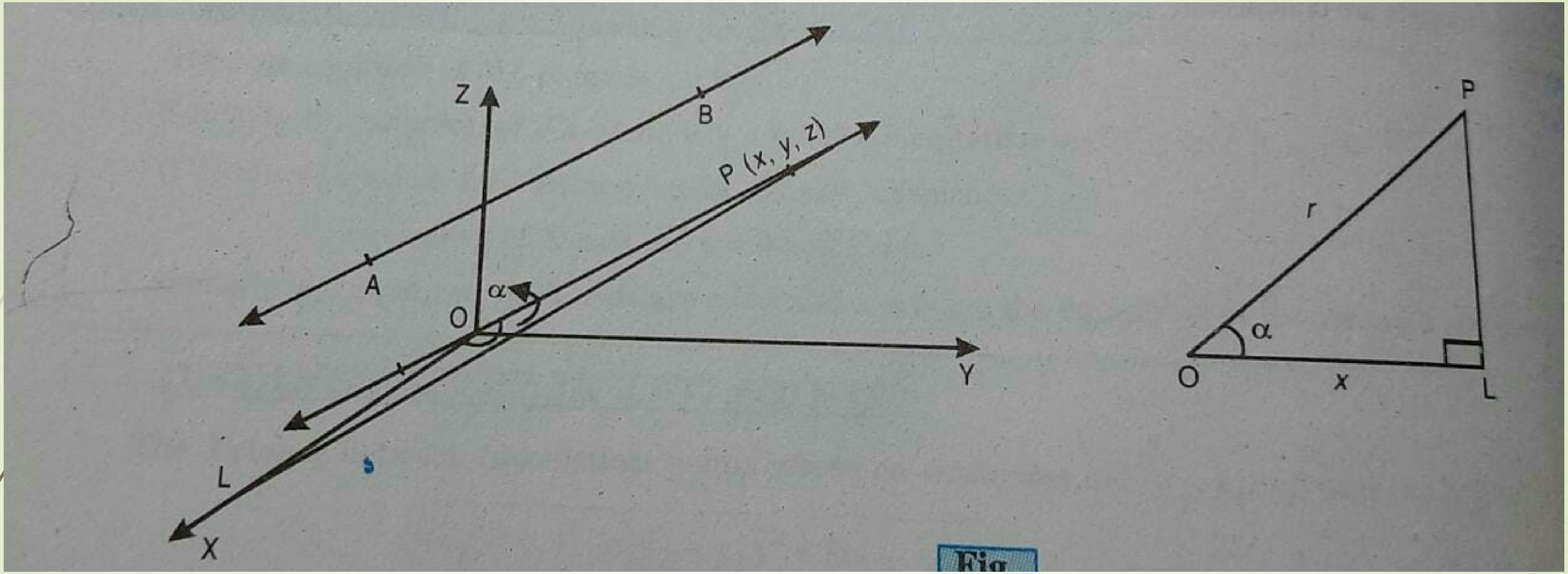
l=cosa= cos90° $= 0$

m=cosβ=cos60° $= \frac{1}{2}$

n=cosγ=cos30° $= \frac{\sqrt{3}}{2}$

Thus, direction-cosines of a given line are 0, $\frac{1}{2}$, $\frac{\sqrt{3}}{2}$.

RELATION BETWEEN DIRECTION – COSINES OF A LINE



- Let OP be a line which makes an angle α, β, γ with positive direction of x,y and
- Z- axis respectively and l , m , n be its direction-cosines.
- Let the coordinate of P be (x, y, z).
- By distance formula,
- $OP=\sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$
- i.e, $OP = \sqrt{x^2+y^2+z^2} = r$ (say)
- From P, $PL \perp ox$
- In a Right angled triangle ΔLOP,
- $\cos\alpha = x/y$

$\Rightarrow l = x/r$

$\Rightarrow x = lr$

Similarly,

$\text{Cos}\beta = \frac{y}{r} \Rightarrow m = \frac{y}{r} \Rightarrow y = mr$

$\cos\gamma = \frac{z}{r} \Rightarrow n = \frac{z}{r} \Rightarrow z = nr$

Now, $x^2 + y^2 + z^2 = l^2 r^2 + m^2 r^2 + n^2 r^2$

$\Rightarrow r^2 = r^2 (l^2 + m^2 + n2)$

$(\because r = \sqrt{x^2+y^2+z^2})$

$\Rightarrow l^2 + m^2 + n^2 = 1$

Which is required relation between direction-cosines of a line.

DIRECTION-RATIOS OF A LINE

Any three numbers, which are proportional to the Direction-cosines of a line, are called the direction-ratios Of the line.

If a, b, c be the direction-ratios of a line.

Let l, m, n be its direction-cosines.

From definition,

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c} = k(say), where\ k \neq 0.$$

=> l = ka, m =kb,n= kc

We know

$$l^2 + m^2+n^2=1$$

$$\Rightarrow \quad k^2\,a^2+k^2b^2+k^2c^2=1$$

$\Rightarrow K^2(a^2+b^2+c^2)=1$

$\Rightarrow k^2=1/(a^2+b^2+c^2)$

$\Rightarrow k=\pm 1/\sqrt{a^2+b^2+c^2}$

Now,

$l=ka=\pm a/\sqrt{a^2+b^2+c^2}$
$m=kb=\pm b/\sqrt{a^2+b^2+c^2}$

$n=kc=\pm c/\sqrt{a^2+b^2+c^2}$

Thus, If a ,b ,c ,be the direction-ratios of a line , then the direction-

Cosines of the line are $\pm a/\sqrt{a^2+b^2+c^2}$, $\pm b/\sqrt{a^2+b^2+c^2}$, $\pm c/\sqrt{a^2+b^2+c^2}$

Where sign should be taken all positive or all negative.

NOTE :- $l^2+m^2+n^2 = 1$ but $a^2+b^2+c^2 \neq 1$

Question : If a line has direction – Ratio's +2, -1, -2. determine its direction cosines.

Soln : Given (a,b,c) = (2,-1,-2)

Now,

$$\sqrt{a^2+b^2+c^2} = \sqrt{2^2+(-1)^2+(2)^2}$$

$$= \sqrt{4+1+4} = 3$$

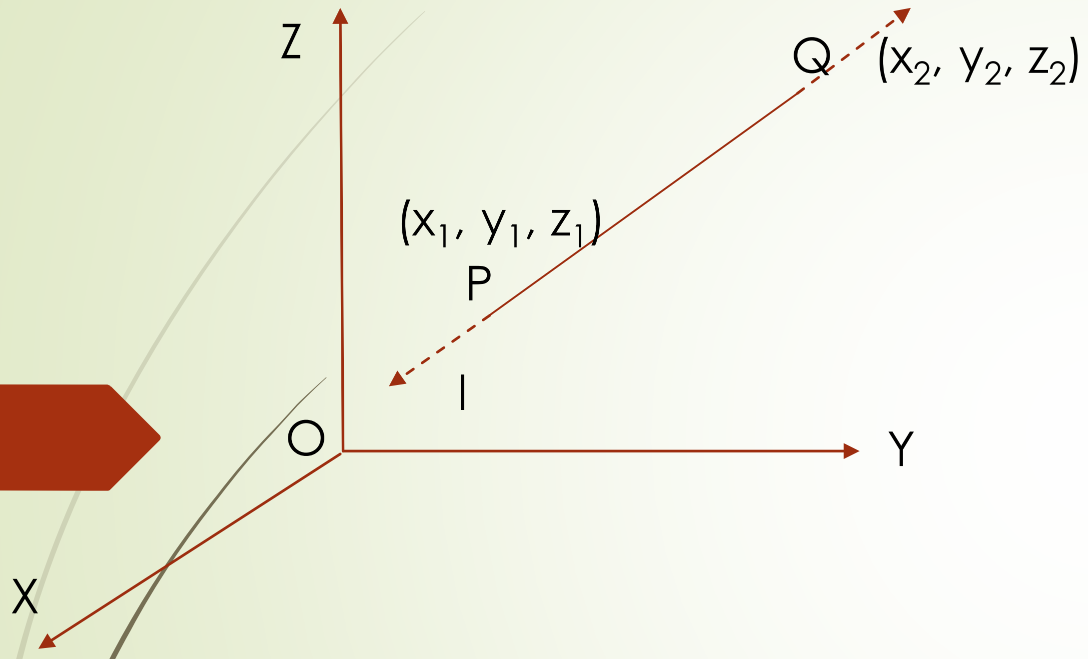
$$l = {}^{a}/_{\sqrt{a^2+b^2+c^2}} = {}^{2}/_{3}$$

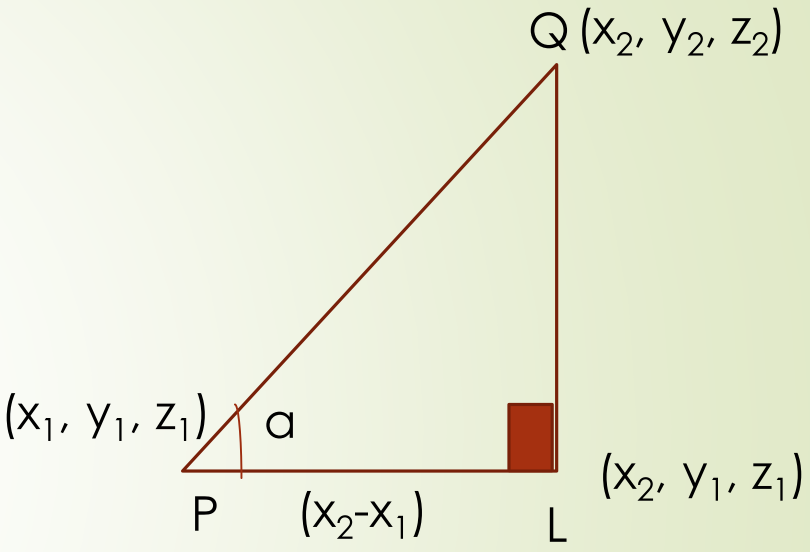
$$m = {}^{b}/_{\sqrt{a^2+b^2+c^2}} = {}^{-1}/_{3}$$

$$n = {}^{c}/_{\sqrt{a^2+b^2+c^2}} = {}^{-2}/_{3}$$

Req. d.c's of a line are 2/3, -1/3, -2/3

Direction – Ratio's of a line joining two points





Let l be a line passing through two given points $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ respectively.

From Distance Formula,

$$PQ = \sqrt{(x_2 - x^1)_2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = r \qquad \text{(say)}$$

$$PL = \sqrt{(x_2 - x_1)^2 + (y_1 - y_1)^2 + (z_1 - z_1)^2} = x_2 - x_1$$

In Right angled triangle Δ PLQ,

$$\text{Cosa} = \frac{PL}{PQ}$$

$$\Rightarrow \text{Cosa} = \frac{(x_2 - x_1)}{PQ} = \frac{x2 - x_1}{r}$$

$$\Rightarrow l = \frac{(x_2 - x_1)}{r}$$

$$\Rightarrow lr = x_2 - x_1$$

Similarly,

$$mr = y_2 - y_1$$

$$nr = z_2 - z_1$$

If a,b,c be the d.r.'s of line joining two points $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ then

$a = lr \Rightarrow a = x_2 - x_1$

$b = mr \Rightarrow b = y_2 - y_1$

$c = nr \Rightarrow c = z_2 - z_1$

Again,

$$l = \frac{x_2 - x_1}{r} = \frac{x_2 - x_1}{PQ}$$

$$m = \frac{y_2 - y_1}{r} = \frac{y_2 - y_1}{PQ}$$

$$n = \frac{z_2 - z_1}{r} = \frac{z_2 - z_1}{PQ}$$

Hence, d.r's of a line passing through two given points P(x1,y1,z1) and $Q(x_2,y_2,z_2)$ are $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$ and its d.c's are $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$ Where $PQ = \sqrt{}(x_2 - x_1)_2 + (y_2 - y_1)_2 + (z_2 - z_1)_2$

Question: Find the d.r's and d.c's of a line passing through two given points A(1,2,-2) and B(3,-4,5) respectively.

Solution: Given points are A(1,2,-2) & B(3,-4,5).

From Distance formula,

AB= $\sqrt{(3-1)2+ (-4-2)2+ (5+2)2}$

$=\sqrt{4+36+49}$

$=\sqrt{89}$

Now,

Direction-Ratios (d.r's) of a given line are 3-1, -4-2, 5+2 i.e, 2, -6, 7.

Direction Cosines (d.c's) of a given line are $\frac{2}{\sqrt{89}}, \frac{-6}{\sqrt{89}}, \frac{7}{\sqrt{89}}$

Question: Find the direction- Cosines of a line, which makes equal angles with the co-ordinate axes.

Solution: Let α, β, γ be the direction- angles (d.a's) of a given line

From Question,

$\alpha=\beta=\gamma$

$\cos\alpha= \cos\beta= \cos\gamma$

$L= m= n= k(say)$

We Know

$l^2+m^2+ n^2 =1$

$k^2+k^2+ k^2 = 1$

$3k^2 = 1$

$K^2 = \frac{1}{3}$

$K= \pm\frac{1}{\sqrt{3}}$

Hence, direction- Cosines of the line are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$.