SUB – MATHS

CLASS- 12th

DETERMINANT

DEFINATION :-

Each square matrix A of order "n" there is associated an expression which has a fixed value is called the determinant of A, denoted by det A or |A| of order n.

$$\text{IF } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots.. & \dots.. & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots.. & \dots.. & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots.. & \dots.. & a_{nn} \end{bmatrix}$$

Then:-

$$\text{Det } A = |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots.. & \dots.. & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots.. & \dots.. & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots.. & \dots.. & a_{nn} \end{vmatrix}$$

value of a determinant of order 1:-

the value of a determinant of a (1 x 1) matrix [a] is defined as |a|= a

value of a determinant of order 2 :-

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (a_{11}a_{22} - a_{12}a_{21})$$

Eg:- $\begin{vmatrix} 6 & -3 \\ 7 & -2 \end{vmatrix}$

$= 6 \times (-2) - 7 \times (-3)$

$= -12 + 21$

$= 9$

Minor of a_{ij} in |A| :-

The minor of an element a_{ij} in |A| is defined as the value of the determinant obtained by deleting the i^{th} row and j^{th} column of |A| and it is denoted by M_{ij} .

Cofactor of a_{ij} in |A|:-

The cofactor C_{ij} of an element a_{ij} is defined as

$$C_{ij} = (-1)^{i+j} . M_{ij}$$

eg. Find the minors and cofactors of the elements of the determinant .

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

to find minor M_{11} of element a_{11} that occurs in 1^{st} row and 1^{st}column. we delete the 1^{st} row and 1^{th} column

$$\therefore M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = (a_{22}\, a_{33} - a_{32}\, a_{23})$$

similarly :-

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = (a_{21}\, a_{33} - a_{23}\, a_{31})$$

$$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = (a_{12}\ a_{33} - a_{32}\ a_{13})$$

now cofactor of a_{ij} is :-

$$C_{11} = (-1)^{1+1}.\ M_{11} = (a_{22}\, a_{33} - a_{32}\, a_{23})$$

$$C_{12} = (-1)^{1+2}\ M_{12} = -\ (a_{21}\, a_{33} - a_{23}\, a_{31}) = (a_{23}\, a_{31} - a_{21}\, a_{33})$$

$$C_{21} = (-1)^{2+1}\ M_{21} = -\ (a_{12}\, a_{33} - a_{32}\, a_{13}) = (a_{32}\, a_{13} - a_{12}\, a_{33})$$

Value of a determinant :-

The value of a determinant is the sum of the products of elements of a row (or a column) with their corresponding cofactors.

Expansion of a determinant :-

Expanding the given determinant by 1st row ,and we have

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$= a_{11}$.(its cofactor) $+a_{12}$(its cofactor)$+a_{13}$.(its cofactor)

$= a_{11}\,.(a_{22}\, a_{33} - a_{32}\, a_{23}) - a_{12}\,.(a_{21}\, a_{33} - a_{31}\, a_{23}) + a_{13}\,.(a_{21}\, a_{32} - a_{31}\, a_{22})$

Note :-1

If we expend a determinant by any row or column using minors ,we following symbols for a determinant of order three.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Note 2:- if a row or a column of a determinant consist of all zero , the value of the determinant is zero .

Properties of determinant :-

1 :- the value of a determinant remains unchanged if its rows and columns are interchanged.

$$\text{Let } A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Interchanging the rows and column of A

$$\text{Then } A' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$= A.$

2 :-If a is a square matrix then $|A'| = |A|$

3 :- If two rows or columns of a determinate are interchanged then the determinant retain its absolute value but its sign is changed.

$$\text{Let } \triangle = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

interchanging any two rows ,say 1st and 3rd row

$$\triangle' = \begin{vmatrix} a_3 & b_3 & c_3 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix}$$

$= -\triangle$

4 :- if any two rows or columns of a determinant are identical then its value is zero.

$$\text{If } A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix}$$

we can see row 1 and row 3 are identical then the value of determinant is

$$|A| = 0$$

5:- For a square matrix A of order n:

$$|KA| = K^n |A|$$

- **Steps to find the value of determinant by elementary transformation :-**

I :- by interchanging i^{th} and j^{th} rows or column

II :- after elementary transformation take common

III :- on expanding

QUEST :- prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

SOL :- let

$$\triangle = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply $R_1 \rightarrow R_1 + R_2 + R_3$

$$\triangle = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking (a+b+c) common from R_1

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_2 \rightarrow c_2 - c_1$ and $c_3 \rightarrow c_3 - c_1$

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

Expanding by R_1

$= (a+b+c)\ [1\{-(a+b+c) \times -(a+b+c) - 0\}]$

$= (a+b+c)\ (a+b+c)^2$

$= (a+b+c)^3$ proved

Area of a triangle in determinant from :-

Area of a triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by,

$$\triangle = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$

- If area of triangle is zero then the three points A.AB.C. are collinear
- $\triangle ABC = 0 \Leftrightarrow$ A,B,C are collinear.

MODEL QUESTIONS

SECTION A

1 MARKS QUESTION

QUESTION 1:- Evaluate the determinant

$$\begin{vmatrix} \text{Sin } 70^0 & -\cos 70^0 \\ \text{Sin } 20^0 & \cos 20^0 \end{vmatrix}$$

SOLUTION :-

$$\begin{vmatrix} \text{Sin } 70^0 & -\cos 70^0 \\ \text{Sin } 20^0 & \cos 20^0 \end{vmatrix}$$

$= \sin 70^0 \cos 20^0 - (-\cos 70^0 \sin 70^0)$

$= \sin 70^0 \cos 20^0 + \cos 70^0 \sin 20^0$

$= \sin (70^0 + 20^0)$

$= \sin 90^0$

$=$ **1 ans.**

QUESTION 2 :- Find the value of (x)

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

SOLUTION:- $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36$

$\therefore$ x = 6 ans.

QUESTION.3:- Find the minors of $\begin{vmatrix} 2 & 3 \\ -4 & 1 \end{vmatrix}$

SOLUTION :- $M_{11} = 1$

$M12 = -4$

$M_{21} = 3$

$M_{22} = 2$

$\therefore M_{ij} = \begin{vmatrix} \mathbf{1} & \mathbf{-4} \\ \mathbf{3} & \mathbf{2} \end{vmatrix}$ **ans .**

QUESTION 4 :- Evaluate :- $\begin{vmatrix} \mathbf{4} & \mathbf{9} & \mathbf{7} \\ \mathbf{3} & \mathbf{5} & \mathbf{7} \\ \mathbf{5} & \mathbf{4} & \mathbf{5} \end{vmatrix}$

SOLUTION :- let

$A = \begin{vmatrix} 4 & 9 & 7 \\ 3 & 5 & 7 \\ 5 & 4 & 5 \end{vmatrix}$

Expanding by R_1

$= 4(5 x5 – 7x4) -9 (3x5 -7x5) + 7 (3x4 – 5x5)$

$= 4 (25 -28) -9 (15 -35) + 7 (12-25)$

$= 4 x(-3) – 9x(-20) + 7 (-13)$

$= - 12 + 180 – 91$

= 77 ans.

SECTION [B]

4 MARKS QUESTION

QUESTION.1:-

$$\begin{vmatrix} \mathbf{a} & \mathbf{a+b} & \mathbf{a+b+c} \\ \mathbf{2a} & \mathbf{3a+2b} & \mathbf{4a+3b+2c} \\ \mathbf{3a} & \mathbf{6a+3b} & \mathbf{10a+6b+3c} \end{vmatrix} = \mathbf{a^3}$$

SOLUTION:-

$$\text{Let } A = \begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - 3R_2$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 0 & a \end{vmatrix}$$

Expanding along C_1

$= a\,[a \times a - 0]$

$= a^3$ proved

QUESTION 2 :- Prove that . $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

SOLUTION :- Let

$$A = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$

$R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

Taking (a-c) and (b-c) common from R_1 and R_2 respectively

$$=(a-c)(b-c)\begin{vmatrix} 0 & 1 & a+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$=(a-c)(b-c)\begin{vmatrix} 0 & 0 & a-b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expanding along R_1 we get

= (a-c) (b-c) [a-b (0-1)]

= (a-c) (b-c) (a-b) (-1)

= **(a-b) (b-c) (c-a) proved.**

QUESTION 3 :- prove that

$$\begin{vmatrix} 1 & x^2 & x^2 \\ x^2 & 1 & x^2 \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

SOLUTION :-

$$\text{Let } A = \begin{vmatrix} 1 & x^2 & x^2 \\ x^2 & 1 & x^2 \\ x & x^2 & 1 \end{vmatrix}$$

Applying $c_1 \to c_1 + c_2 + c_3$

$$= \begin{vmatrix} 1+x^2+x^2 & x & x^2 \\ 1+x^2+x^2 & 1 & x \\ 1+x^2+x^2 & x^2 & 1 \end{vmatrix}$$

Taking $(1 + x^2 + x^2)$ common from c_1

$$= (1 + x^2 + x^2) \begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$= (1 + x^2 + x^2) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 - x & x - x^2 \\ 0 & x^2 - x & 1 - x^2 \end{vmatrix}$$

Expanding along c_1

$= (1 + x + x^2) \quad [1 \{(1 - x)(1 - x^2) - (x - x^2)(x^2 - x)\}]$

$= (1 + x^2 + x^2) \quad [1 - \not{x^2} - x + \not{x^3} - \not{x^3} + \not{x^2} + x^4 - x^3]$

$= (1 + x^2 + x^2) \quad [1 - x - x^3 + x^4]$

$= 1 + \not{x} + \not{x^2} - \not{x} - \not{x^2} - x^3 - x^3 - \not{x^4} - \not{x^5} + \not{x^4} + \not{x^5} + x^6$

$= 1 - 2x^3 + x^6$

$= (1 - x^3)^2$ proved.

SHARMISTHA MANDAL

PGT (MATHEMATICS)

PROJECT GIRLS +2 HIGH SCHOOL,

POTKA ,EAST SINGHBHUM